A new numerical model to study isolated rock blocks around underground excavations taking into account in-situ stresses

Rima Ghazal, Faouzi Hadj Hassen, Michel Tijani

ARMA 11-251

A new numerical model to study isolated rock blocks around underground excavations taking into account in-situ stresses

Ghazal, R. Hadj-Hassen, F. and Tijani, M.
Centre de Géosciences, Mines-ParisTech, Fontainebleau, France

This paper was prepared for presentation at the 45th US Rock Mechanics / Geomechanics Symposium held in San Francisco, CA, June 26–29, 2011.

ABSTRACT: When an underground excavation is cut in a discontinuous rigid rock mass, instabilities may occur mainly due to block failure. Isolated rigid block methods were developed since the 80's to locate critical blocks and evaluate their stability but have major drawbacks: ignoring in-situ stresses, mechanical behavior of joints and rotational movements. Other methods included those variables improperly and were limited to simple cases. This paper presents a critical review of previous approaches then a more complete model to study isolated rock blocks is proposed. It is based on the fact that stresses on the block faces are known before excavation and once a face is freed, the block moves as a rigid body in translation and rotation. Applying equilibrium and rock joint behavior equations, the stresses on the faces after excavation can be calculated and stability evaluated using a Mohr-Coulomb criterion. Any block geometry can be studied by partitioning the block faces into simple elements. Numerical integration is done on elements using Gauss points. The method is applied on a case study and comparisons are made with other simplified methods. Finally, a parametrical analysis shows the important influence of in-situ stresses and joint stiffnesses on the block's stability.

1. INTRODUCTION

When underground excavations are made in rigid rock masses intercepted by several discontinuities, blocks may form at the free surface and present the risk of sliding or falling into the open space causing damage. Modeling such phenomena is an essential requirement in order to predict a degree of instability and evaluate the support needed.

Adopting a continuum model without discontinuities is not appropriate in this case since the main deformation occurs due to the displacements about the joints rather than to the deformation of the rock matrix.

Furthermore, applying a complete discontinuous method, including all joints, is computationally hard because of the complexity of the three dimensional geometry. Additionally, the uncertainties concerning the distribution of joint sets require performing multiple simulations in order to cover all the possibilities.

From the necessity to overcome this complexity, came the idea of studying separately only the blocks formed at the surface of the tunnel. Supporting the unstable blocks is assumed to assure stability for all the rock mass. This approach is simple and can provide the engineer with an easy tool to evaluate the stability of the excavation or to choose its optimum direction.

The main assumptions of isolated rock block methods are:

- The rock mass is infinitely rigid and any displacement takes place only along the discontinuities, supposed to be perfectly planar.
- While studying one rock block, the rest of the rock mass is considered to be rigid and continuous. Interaction between several blocks is not taken into account.

The generation of blocks can be made by studying all the combinations of discontinuities that may form removable blocks at the surface of the tunnel. This approach does not require information of spacing between discontinuities or their exact location (ubiquitous approach). Another approach consists of generating all blocks by using distribution of joint sets or introducing joints one by one (specific approach). This article does not consider the problem of generation of the rock blocks in the rock mass. It focuses only on the stability analysis of a rock block once it is generated by whatever method.

Isolated rock block methods have been widely used. But despite the enormous simplification of ignoring the interaction with other blocks, these methods still lack the ability to give a rigorous representation of the block behavior.

The following paragraph gives a review of the principal defaults of the methods so far used. Then, a complete model considering the real behavior of a block as a rigid body translating and rotating is presented. In this model , the initial stresses on the joints are relaxed with the block's displacement while respecting equilibrium equations. Despite its simplicity, such a model has never been addressed in literature.

2. REVIEW OF ROCK BLOCK STABILITY METHODS

2.1. Classical 'Keyblock' method (limit equilibrium method)

The method was first developed by 'Goodman and Shi' and 'Warburton' and is a limit equilibrium method [1,2].

In addition to the general assumptions of isolated block theory, stresses acting on the block faces are supposed to be uniformly distributed and are replaced by one 'reaction' force vector on each face. These are calculated ignoring in-situ stresses in the rock mass. The only known forces acting on the block are its weight and possibly another force resulting from water pressure and support. Once the reaction forces determined the block is declared to be unstable if these forces exceed the forces that will cause limit equilibrium.

Indeterminate problem and assumptions:
Equilibrium equations are not enough to determine the 'reaction' forces. Even for the simplest three-dimensional block, a tetrahedral block, the number of unknowns is 9 (three dimensional reaction force on each face), while the number of equilibrium equations for forces and moments is 6.

To solve that indetermination, the 'Keyblock' approach postulates that reaction forces can only occur on two faces, one face or none, reducing the number of unknowns to a maximum of 6. It follows that in order to determine which faces are the ones 'carrying' the reactions, a pre-analysis should be made to find out how the block will move. If only one or two faces can 'carry' a force it is assumed that other faces with null forces should detach from the rock mass.

Kinematical analysis:
Consequently, the only possible movements of the block that can be considered to make the problem solvable are sliding on one face, two faces or falling. Although this assumption is understandable for a tetrahedral block, having three contact faces, it does not represent the real situation when a block is an arbitrary polyhedron.

The mode of movement needs to be studied with predefined rules considering only the geometry of the block and the direction of the 'active' force (weight and possibly water forces or another exterior force).

If $\vec{A}$ is the 'active' force acting on the block, $\vec{n}_i$ the interior normal to the block face 'i', $\vec{s}_i$ the possible sliding direction on face 'i' and $\vec{s}_{ij}$ the possible sliding direction on the intersection of faces 'i' and 'j', these rules are as follows :

Falling:

$$\forall i\ \vec{A}.\vec{n}_i > 0 \tag{1}$$

For sliding on one face i:

$$\vec{A}.\vec{n}_i \le 0 \text{ and } \forall j \ne i\ \vec{s}_i.\vec{n}_j > 0 \tag{2}$$

Sliding on two faces:

$$\forall k \ne i \text{ et } k \ne j,\ \vec{s}_{ij}.\vec{n}_k > 0\ \ \vec{s}_i.\vec{n}_j \le 0\ \ \vec{s}_j.\vec{n}_i \le 0 \tag{3}$$

The direction of movement is the same as the active force if the block is falling. It is the projection of the active force on the sliding face if sliding on one face and it is the same as the intersection of two faces if sliding on two faces. (See "Fig. 1.")

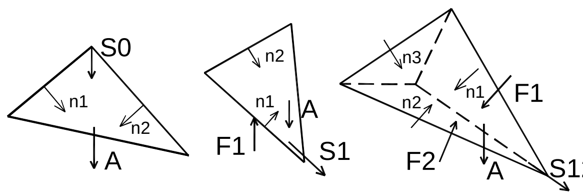

Fig. 1. Modes of displacement in the 'keyblock' theory

Mechanical behavior, a consequence of kinematical behavior:

The determination of forces acting on the faces depends on the kinematical analysis. That separation renders the problem solvable but there is no rigorous justification for it. Once the forces acting on the faces are determined, a factor of safety 'SF' is calculated:

- Falling wedge: Forces are null and SF=0
- Sliding on one or two faces: Forces are calculated using equilibrium equations and assuming that the shear forces have the same direction as sliding direction. If F_n and F_t are the norms of normal and shear forces respectively, c the cohesion, S the surfaces of sliding and ϕ the friction angle,

$$SF = \left(c\,S + F_n \tan(\phi)\right) / F_t \tag{4}$$

Further remarks:
Studied using that approach, the problem of the block at the surface of a tunnel is equivalent to the problem of someone putting a block at its place in a mold representing a tunnel and seeing how it will move with the additional simplification of ignoring rotation. It is an analysis only based on geometry and does not represent the real conditions underground. The successive

simplifications adopted and the technique of dividing the problem to kinematical and mechanical analysis serve to make the problem solvable but do not contribute to model rigorously the behavior of the block.

Some studies aimed to validate 'Keyblock' theory using field case history or physical models. In both cases the in-situ stresses were too low to validate the method for underground conditions [3,4]. Additionally, it was shown that 'Keyblock' theory gives wrong results when complex behaviors occur, such as rotation associated to translation [3].

2.2. 'Keyblock' improvements

Most of 'Keyblock' method improvements were concerned with geometrical aspects without reconsidering the mechanical stability evaluation of a single block. In fact, since the conception of 'Keyblock' theory, many softwares were developed on its basis. The only feature that distinguishes one from the other is the method used for generating the blocks [5-12].

Some approaches looked back at the stability evaluation considering probabilities. A method to identify the most critical block based on probabilities of joint intersection was developed [13]. Also, some studies tried to generalize the method to a group of blocks in 2D [14].

The only methods that tried to integrate in-situ stresses and rotation for a better modeling of the block stability are explained hereafter.

Including in-situ stresses:

'Keyblock' method, as explained above, is more adapted for problems where in-situ stresses are low. Aware of that limitation, 'Unwedge' software developers included in-situ stress analysis to the classical methodology [12,15]. Supposing the initial stresses to be known before excavation, a 'boundary element model' is used to calculate the redistribution of stresses after excavation. This model is continuous ignoring the discontinuities, i.e. the block faces are only surface geometries. The normal forces acting on the wedge faces are calculated from the integration of stresses obtained from the continuous model. These normal forces are added to the 'active' force then the usual kinematical analysis is performed. Unlike the classical kinematical analysis, the shear strength is calculated on all faces and not only on the sliding faces. The methodology can be criticized based on the following points:

- Actual normal stresses acting on the wedge faces are different from the normal stresses calculated from a model where discontinuities are not included.
- Only one part of the information from stress analysis is kept: shear stresses are not considered during the active force calculation. The only justification is that, if they were added, equilibrium necessarily implies that the active force would be null and any further analysis would be impossible using the 'Keyblock' methodology.
- If the continuum model really gave accurate information of the stresses acting on the faces, it would be more logical to analyze the stability of the wedge by checking if the criterion (Mohr-Coulomb for example) is respected on all faces.

Consequently, this approach is a palliative to the insufficiencies of block theory but does not present a rigorous well justified scientific approach.

Including rotation:

Rotational movements can intervene to stabilize the block or on the contrary to constrain its movement. These are not included in the classical block theory analysis. Some authors have developed kinematical rules to determine the mode of possible rotation and to evaluate stability based on these phenomena [16-18]. However, among all the available softwares based on 'Keyblock' theory, none has included rotational analysis. In fact, the complexity of this analysis renders it difficult to integrate in the 'Keyblock' algorithm.

2.3. Relaxation models for falling blocks

2D relaxation methods:

Many authors adopted analytical solutions for the stability of blocks in the roof of underground excavations considering the effect of stresses and joint behavior [19-24]. However, the approaches are generally limited to simple 2D cases and present a lot of drawbacks.

The general procedure was first developed by Hoek for a symmetric block on the roof of a horizontal excavation [19]. It consists of a two stages analysis, as illustrated in "Fig. 2":

(i) The joints are infinitely stiff while the surrounding rock mass is homogeneous, isotropic, linearly elastic transmitting a horizontal force H_0 to each face. The block weight is not supposed to act at this phase.

(ii) The rock mass is infinitely rigid whereas the joints are deformable. The block's weight is supposed to act as well as any additional force R. The displacement of the block is calculated by relaxing the joint forces so as to restore the equilibrium. The normal force N on a joint vary linearly with the normal displacement U_n according to $N=N_0-K_nU_n$. The shear force also vary linearly with shear displacement U_t and can be calculated by $T=T_0-K_tU_t$. A pull out force P_l that causes limit equilibrium on the faces serves to evaluate the stability of the block. The coefficient (R+Pl)/W is used as the factor of safety

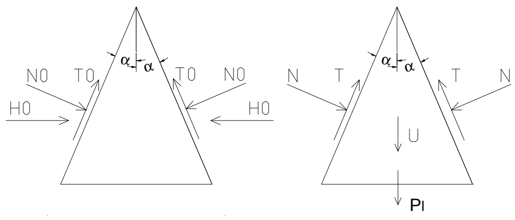

Fig. 2. Two-stage relaxation method for a 2D block

During calculation, the stresses are supposed to be uniform on the faces of the block. Hence, yield occurs uniformly on all the point of both faces (or one face if the block is not symmetric). The displacement of the block takes place vertically downward in the second stage.

The improvements and general limitations are explained hereafter:

Crawford and Bray (1983) showed by numerical modeling that the presence of a vertical stress has a destabilizing effect on the wedge [20]. However, in the analytical procedure adopted, this vertical component acting on the face has to be null at the first stage. That is due to the necessity of verifying equilibrium equations while considering one force per face.

Some authors calculated the initial forces acting on the wedge (N_0 and T_0) integrating stresses from an elastic continuous model : Elseworth (1986) considered an initial hydrostatic stress field around a circular opening, [21]; Sofianos et al. (1999) complicated the case by assuming non hydrostatic stress field [22]; Nomikos et al. (2002, 2006) considered an inclined stress field [23, 24]. In all cases, one force per face is considered (as if considering uniform distribution of stresses on each face). Comparison made by the authors with a 'Udec' model showed a similitude of the pull out resistance for the cases of a horizontal excavation roof. However, for the cases of circular excavation, the analytical model overestimates the stability. This is due to partial yield of the joint face that cannot be taken into account when considering one force per face.

On the other hand, integrating the initial stresses from a continuous model is not well justified [21, 24]. (See 'section 3.5' for comparison of stresses on the block between continuous and discontinuous models). Furthermore, the weight is not considered to have any influence on the distribution of stresses at the first stage. It is only by adding an additional force to cause yield that joints are solicited in this method.

Concerning kinematics, rigid body displacement is modeled without taking into account the rotation component, not even for the inclined stress field or for the non symmetric wedges. Let's note that this omission is directly linked to the simplification of one force per face. In this case, all points belonging to one face should have the same displacement vector.

Finally, all the methods described above are in 2D and limited to simple triangular roof blocks. Furthermore, the principle of dividing the analysis in two steps, where the weight does not affect the initial stresses, is not representative of the real block's behavior during excavation.

3D relaxation method:
This method is an extension of previous 2D relaxation methods. It uses the same two steps analysis but, unlike previous methods, the block's faces are meshed [25]. Stresses at the first stage are integrated from a continuous model and shear stresses are modified when the Mohr-Coulomb criterion is not respected. By doing so, the equilibrium of the block is violated at the first stage and all the rest of the analysis becomes inaccurate. At the second stage, a vertical downward displacement of the block is assumed and a corresponding pull out force calculated. This pull out force is not representative of the block state of stability since one part of it contributes to restore the equilibrium that was initially violated.

2.4. Brady and Brown limit equilibrium method
This method studies a tetrahedral block whose summit is vertically projected on its base [19]. The block's displacement is assumed to be vertically down and joint behavior is ignored. The indetermination of forces on the faces is resolved by the following procedure:

Normal forces on the joints are integrated from a previous elastic analysis considering a deformable rock with no discontinuities. By assuming that all faces yield at the same time a pull out force is calculated. At each face, shear forces causing yield are supposed to have the same direction as the bisecting line of the face. This assumption is not justified.

Although this method takes account of in-situ stresses, it is limited to simple block cases, calculates stresses using a continuous model, and does not consider the effect of joint behavior.

2.5. Conclusion
Despite the enormous simplification of studying one block at a time, considering all the rest of the rock mass to be rigid and continuous, none of the previous methods is a complete mechanical analysis. All are based on successive assumptions to be able to solve the indetermination of forces on the block faces: the 'Keyblock' method divides the analysis into kinematical and mechanical analysis considering only simplified displacements and ignoring in-situ stresses; 'relaxation methods' include the rock joint behavior but study only the simple cases of roof block displacing vertically downward; all approaches including stresses calculate

them using a continuous model where the presence of joints is ignored. From those limitations comes the necessity to develop a more complete method that takes into account the effect of in-situ stresses and models accurately the block's translational and rotational movement.

3. NEW DEVELOPED METHOD (ISOBLOC)

The new method studies the block behavior during the process of excavation. Before excavation, stresses in the rock mass and on the block faces are supposed to be known. After excavation, the free faces are distressed causing the block to move. At the contact faces, stresses change according to joint's behavior laws.

The new method adopts the same general assumptions as classical isolated rock block methods: considering a rigid rock and studying one block at a time. However, unlike 'Keyblock' methods, displacement is not predetermined by a kinematic analysis. The indetermination that comes from using only equilibrium equations is solved by considering the joint behavior. Compared to relaxation methods, it is more general and accurate. The analysis leads to solving a linear system whose only unknowns are the displacement vector of a reference point and the rotation vector of the block.

3.1. *Theoretical study*

Equilibrium between initial and final stages:

Considering a block (Ω) located at the surface of a future excavation, it is limited by a boundary ∂Ω=∑J+∑L where ∑J designates the joint boundaries and ∑L the boundary to be excavated.

At the initial stage, before excavation, the stress tensor $\underline{\underline{\sigma_0}}$ is known at every point of the rock mass. The block is at equilibrium under volume forces $-\vec{\nabla}\underline{\underline{\sigma_0}}$ (i.e. its weight if stresses vary linearly with depth) and surface forces $-\underline{\underline{\sigma_0}}\,\vec{n}$ (integration of stresses on the block faces).

The process of excavation is modeled by diminishing the initial surface forces acting on the 'future' free faces. During excavation the block is maintained at equilibrium. Volume forces remain the same and surface forces vary linearly with the displacement. At the end of excavation, the stresses are null at any point of the free faces or equal to $(-p_S.\vec{n})$ if a reinforcement (p_s) is applied.

We designate by $\Delta\vec{\sigma}$ the variation of the stress vector related to the local displacement of the point considered and by p_f a possible additional water pressure due to a fluid contained in the joint.

At any point $\vec{x}$ of the block's boundary, the final stress state, after excavation, is as follows:

$$\vec{\sigma}(x) = \begin{cases} -p_S\,\vec{n} & if\ \vec{x}\in\sum L \\ \underline{\underline{\sigma_0}}\,\vec{n}+\Delta\vec{\sigma}-p_f\vec{n} & if\ \vec{x}\in\sum J \end{cases} \quad (5)$$

Subtracting the equilibrium equations of forces and moments, between the stages after and before excavation, leads to the following equations:

$$\int_{\partial\Omega}\left(\Delta\vec{\sigma}-c\underline{\underline{\sigma_0}}\vec{n}-p\vec{n}\right)dS=\vec{0} \quad (6)$$

$$\int_{\partial\Omega}\vec{x}\wedge\left(\Delta\vec{\sigma}-c\underline{\underline{\sigma_0}}\vec{n}-p\vec{n}\right)dS=\vec{0} \quad (7)$$

where c(x)= 1 and $p=p_s$ if x∈∑L

c(x)= 0 and $p=p_f$ if x∈∑J

Joints behavior : expression of $\Delta\vec{\sigma}$:

At any point $\vec{x}$ of the block joint faces, normal and shear stresses vary linearly respectively to its normal and shear displacements. The joint behavior is expressed in "Eq.(8)" and "Eq.(9)".

$$\Delta\sigma_n=-K_nU_n \quad (8)$$

$$\Delta\vec{\tau}=-K_t\vec{U}_t \quad (9)$$

Where at the point $\vec{x}$ considered:

- K_n and K_t are normal and shear stiffness respectively.
- $U_n=\vec{U}.\vec{n}$ and $U_t=\vec{U}-\left(\vec{U}.\vec{n}\right).\vec{n}$ are the normal and shear relative displacements and $\vec{U}$ the vector of relative displacement
- $\Delta\sigma_n=\Delta\vec{\sigma}.\vec{n}$ and $\Delta\vec{\tau}=\Delta\vec{\sigma}-\Delta\sigma_n\vec{n}$ are the normal and shear stress variations.

By combining these equations, the variation of stresses at one point of a joint can be expressed by "Eq.(10)".

$$\Delta\vec{\sigma}=-\underline{\underline{H}}\vec{U} \text{ with } \underline{\underline{H}}=K_t\underline{\underline{I}}+(K_n-K_t)\vec{n}\otimes\vec{n} \quad (10)$$

The elastic behavior of the joint is adopted to actually permit the determination of stresses assuming the block to be stable at the final stage. Nevertheless, stresses will be checked for compatibility with joint strength and the stability of the block will be judged based on real joint's contact laws (see "section 3.2").

Rigid body movement:

The block moves as a rigid body, thus the displacement of any point of the block can be expressed as a function of two vectors. If $\vec{U}_0$ is the displacement vector of a reference point belonging to the block (here the center of mass is adopted) and $\vec{W}_0$ the vector of rotation of the block, $\vec{U}(x)$ at any point $\vec{x}$ of the block is expressed by "Eq.(11)".

$$\vec{U}(x) = \vec{U}_0 + \vec{W}_0 \wedge \vec{x} \qquad (11)$$

To simplify numerical calculation, the cross product $\left(\vec{W}_0 \wedge \vec{x}\right)$ is replaced by $\underline{\underline{r}}\vec{W} = \vec{W}_0 \wedge \vec{x}$ where $\underline{\underline{r}}$ is the rotational tensor of $\vec{x}$

$$\underline{\underline{r}} = rot(\vec{x})$$

Solving the system:
Combining "Eq.(2)" and "Eq.(3)" (equations of equilibrium), "Eq.(4)" and "Eq.(5)" (equations of joint behavior) and "Eq.(11)" (rigid body displacement equation), we get the following system:

$$\begin{cases} \underline{\underline{A}}\vec{U}_0 + \underline{\underline{B}}\vec{W}_0 = \int_{\partial\Omega} \left(-c\underline{\underline{\sigma_0}}\vec{n} - p\vec{n}\right) dS \\ \underline{\underline{B}}^T\vec{U}_0 + \underline{\underline{C}}\vec{W}_0 = \int_{\partial\Omega} \vec{x} \wedge \left(-c\underline{\underline{\sigma_0}}\vec{n} - p\vec{n}\right) dS \end{cases} \qquad (12)$$

where

$$\underline{\underline{A}} = \int_{\partial\Omega} \underline{\underline{H}}\, dS \; ; \underline{\underline{B}} = \int_{\partial\Omega} \underline{\underline{H}}\,\underline{\underline{r}}\, dS \; ; \; \underline{\underline{C}} = \int_{\partial\Omega} \underline{\underline{r}}^T\, \underline{\underline{H}}\,\underline{\underline{r}}\, dS$$

Let's note that, in the absence of pressure forces of due to water or reinforcement, the second terms of "Eq.(12)" are equal to the initial forces and moment acting on the free face before excavation.

The only unknowns of this linear system are the six components of displacement and rotation of the reference point $\left(\vec{U}_0, \vec{W}_0\right)$

3.2. *Assessing stability*

Once the linear system "Eqs.(12)" is solved and $\left(\vec{U}_0, \vec{W}_0\right)$ determined the displacement at any point of the block can be calculated using "Eq.(11)" and the stress vector deduced.

The Mohr-Coulomb criterion is adopted for stability evaluation, "Eq.(13)"

$$\left\|\vec{\sigma}_t\right\| \le -\sigma_n\, tan(\phi) + c \quad \text{with}\; \sigma_n \le 0 \qquad (13)$$

$\vec{\sigma}_t$ and σ_n are respectively the shear stress vector and compressive normal stress on a considered point; ϕ is the friction angle and c the cohesion.

If "Eq.(13)" is verified at all points of the block, this means that the stresses stay in an elastic state and the block is judged to be stable. If yield occurs at some point of the block, the block is considered unstable and a support pressure is calculated so as to stabilize the most critical point.

Various stability factors can be evaluated:

- Required friction angle

This angle is evaluated only on points where compression occurs. It represents the friction angle required to have stability without cohesion or a support pressure. The value retained is the maximum value of "Eq.(14)" calculated on all points :

$$\phi_{required} = a\,tan \frac{\left\|\vec{\sigma}_t\right\|}{-\sigma_n} \quad \text{with}\; \sigma_n \le 0 \qquad (14)$$

A safety factor can also be calculated

$$SF = \frac{-\sigma_n\, tan(\phi) + c}{\left\|\vec{\sigma}_t\right\|} \qquad (15)$$

- Minimum support pressure

The idea is to determine the necessary minimum support pressure to apply uniformly, normally to the free boundary of the block, so as to respect the Mohr-Coulomb criterion on all points. This notion is more significant from an engineering point of view than a safety factor calculation. Practically, the stress vector $\vec{a}$ caused by the only application of a unity pressure, uniformly distributed at the free boundary, is calculated at every point of the block. Thus, because of linearity, the application of a pressure 'p_s' results in a state of stress

$$\vec{\sigma}_p = \vec{\sigma} + p_s\,\vec{a} \qquad (16)$$

where $\vec{\sigma}$ is the stress vector without support at a given point.
Then "Eq.(13)" is applied and the minimum required pressure to have stability can be deduced. The value of p_s adopted is the maximum of values obtained to stabilize all points.

3.3. *Effect of initial stresses and stiffnesses*

The variation of initial stresses has a linear effect on the resultant stresses on the block faces. The effect of the initial stress ratio K_0 is analyzed in "section 4".

Concerning joint stiffnesses, it can be demonstrated mathematically that it is only the factor K_n/K_t that affects the results of final stresses and not the values of K_n or K_t taken separately. The augmentation of that factor has a destabilizing effect on the block as will be shown in "section 4".

Ranges of stiffnesses to adopt:
Many authors performed laboratory and field tests on rock blocks to evaluate the joints stiffnesses. It was

shown that these values vary a lot with different parameters: the nature of the joints (filled or unfilled), the nature and thickness of the filling, the dimension of the surface of contact and the normal stresses applied on the joints.

Concerning shear stiffness, a multitude of tests performed in literature and exposed by Barton and Bandis [26] show that, for a range of normal stresses between 1 and 10 MPa and block length between 1 to 10 m, the shear stiffness vary between about 10 MPa/m and 1000 MPa/m. These values are in accordance with those found by Rechitskii [27].

On the other hand in [27], normal stiffness was evaluated as the ratio of the maximum normal stress to the integral closure of the joints when they are compressed from a null normal stress to a maximum normal stress. It was shown that it varies between about 30 and 10000 MPa/m.

Concerning stiffnesses ratio, for similar tests under the same normal stresses K_n/K_t varied from 2 to 123. The highest ratios were obtained for the unfilled joints [27]. Thus in this article, the following numerical applications adopt ratios of K_n/K_t between 1 to 100.

3.4. Numerical integration

Solving the system of "Eq.(12)" requires the calculation of surface integrals. Analytical integration is a hard task especially when the geometry is complicated. Therefore numerical integration is adopted by partitioning the surfaces into elements of simple geometries and using Gauss points. The global integral of a function over a surface is replaced by the sum of integrals over the elements.

$$I = \int \varphi(\vec{x}) = \sum_{i=1}^{n} \int_{\Sigma i} \varphi(\vec{x}) \qquad (17)$$

Elements that can be used in the partitioning

- Triangular elements with 3 or 6 nodes
- Rectangular elements with 4, 6, 8 or 9 nodes
-

All data of the problem (initial stresses, stiffnesses, friction angle etc.) are defined on the elements nodes and interpolated to Gauss points of each element using known interpolation functions Ni.

$$\varphi(\vec{x}_j) = \sum Ni(\xi,\eta)\varphi(\vec{x}_i) \qquad (18)$$

Then, for each element $\sum i$ the integration of function φ is done using its Gauss points as follows:

$$\int_{\sum i} \varphi(\vec{x}) = \sum_{j=1 \to nj} J\omega_j \varphi(\vec{x}_j) \qquad (19)$$

(ξ_j,η_j) and ω_j are the reduced coordinates and weights of the Gauss points of one element.

$\vec{a} = \partial_\xi \vec{x} \wedge \partial_\eta \vec{x}$ and J the Jacobean $J = \|\vec{a}\|$

The partitioning is done for the only purpose of permitting the integration over complicated surface geometries. It does not affect the accuracies of results, the calculation being linear.

Furthermore, the surface meshing is not constrained to respect the rules of coherence of a classical finite element meshing.

Practically, the general integral of a function over all elements is done by summation over all Gauss points of the elements.

Consequently, by using this technique any block geometry, with any number of faces, can be studied. It is only necessary to partition the polygonal faces of the block into triangular or rectangular elements and to define the different data information on the nodes.

This new approach exposed above will be given the name ISOBLOC in the following sections.

3.5. Validation of stress distribution (2D)

The algorithm of ISOBLOC was also developed in 2D to allow easy comparison with finite element model considering a deformable rock mass. To do so an example of a block at the surface of an underground opening is treated using the following methods:

(i) Finite element model with deformable rock and the presence of joints, (VIPLEF finite element code is used, [28]).
(ii) ISOBLOC model with rigid rock mass and joints.
(iii) Finite element model with deformable rock but without joints, (continuous model).

The rock mass modulus used in deformable models is E=100000 MPa. The vertical stresses are isotropic and vary linearly with depth due to gravity with a rock mass density ρ=2000 Kg/m^3. The excavation is at a depth of 500 m. The joints are linearly elastic with shear stiffness K_t=1000 MPa/m and normal stiffness K_n=1000MPa/m or K_n=10000MPa/m.

First, a comparison of principal stress distribution is made between model (i) and model (ii). 'Fig. 3' shows the rotation of principal stresses close to the discontinuities. It appears clearly that, adopting a continuous model for the calculation of stresses, like in classical methods, is inaccurate. Furthermore, when the elastic modulus of the rock mass increases (the rock is more rigid), it is the presence of discontinuities that imposes the stress distribution more than the form of the excavation.

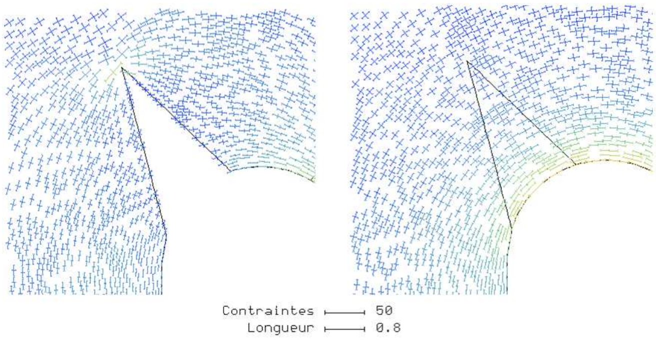

Fig. 3.Stress distribution for a continuous model (right) and a discontinuous model (left).

On the other hand, normal and shear stresses at the faces of the block are compared between the three models. The results of ISOBLOC (ii) are the closest to the exact method (i). To make the results comparable with the 'Keyblock' method the parameter of minimum friction angle required for stability is represented in "Fig. 4". Cohesion of joints is considered null.

Finally, "Fig. 5" shows the movement of the block for the case of K_n/K_t=1 and K_n/K_t=10 respectively using the exact method and the ISOBLOC method. Results of ISOBLOC and the exact method are quite similar. ISOBLOC simulates well the movement of the block even when rotation occurs. The rotation of the block is more apparent for small values of K_n/K_t (in this example for K_n/K_t=1). However, using the 'Keyblock' method, the rotation is ignored. According to the kinematical analysis, the block slides on the critical joint as shown in "Fig. 5". Thus the 'Keyblock' method does not represent well the movement of the block when rotation occurs (here for the case K_n/K_t=1)

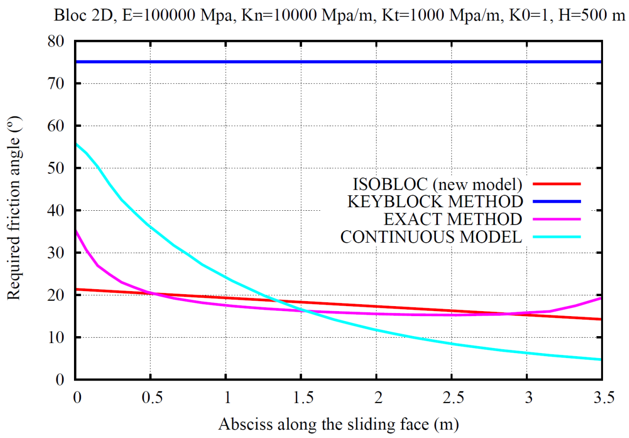

Fig. 4. Comparison of minimum required friction angle along a block's face using different approaches.

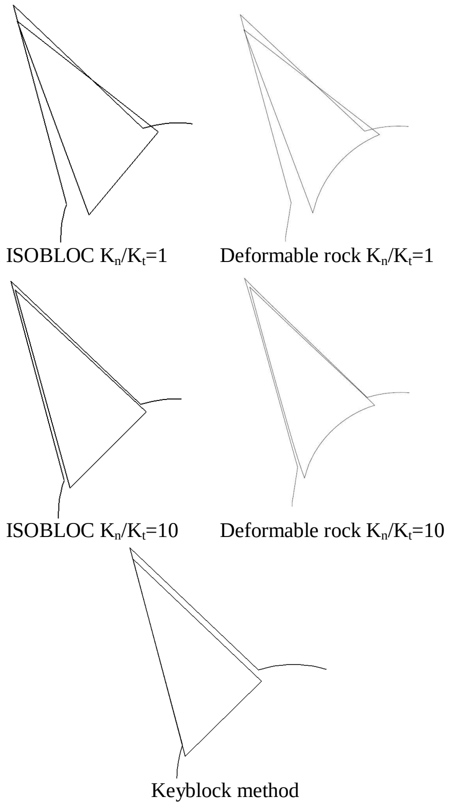

Fig. 5. Displacement of the block according to ISOBLOC, FEM deformable model and Keyblock method.

4. APPLICATION AND PARAMETRICAL STUDY

A 3D bloc formed by the intersection of 3 discontinuities at the roof of a cylindrical excavation is analyzed with ISOBLOC. Then the block is rotated to be able to study different configurations of the orientation of joints and direction of the principal stresses. Comparisons are made with classical 'Keyblock' method.

The initial configuration of joints is shown in 'Table. 1'

Table 1. Structural properties

	Dip (°)	Dip direction (°)
J1	60	0
J2	60	120
J3	60	240

The joints are studied for variable stiffness ratios K_n/K_t=1, 10, 100. The tunnel has a circular area of 10 m diameter, a plunge of 0° and a trend of 0 °. (X,Y,Z) is a direct reference axis where Y is the vertical axis. The origin is set on the tunnel axis. Principal stresses are oriented according to the reference system. Rock mass density ρ=2500 Kg/m^3. H_0=200 m is the depth of the tunnel axis measured from the surface. Initial stresses acting before excavations are as follows:

$$\sigma_{0yy} = \rho g(Y - H_0);\ \sigma_{0xx} = K_{01}\sigma_{yy};\ \sigma_{0zz} = K_{02}\sigma_{yy}$$

The geometries of the blocks studied (same block rotated at 0°, 45° and 90° around the tunnel axis Z) are show in “Fig. 6”.

Block 1:

Comparison of the required friction angle for stability, for different values of K_n/K_t, with respect to the initial stress ratio $K_0=K_{01}=K_{02}$ is shown in the “Fig. 7”. Let’s note that the limit values of K_0 were chosen so that initial stresses are compatible with the Mohr-Coulomb criterion.

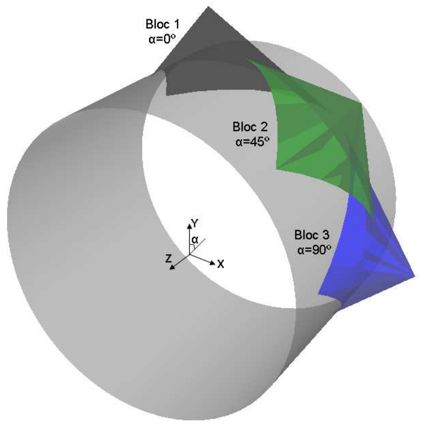

Fig. 6. Geometrical representation of the blocks studied

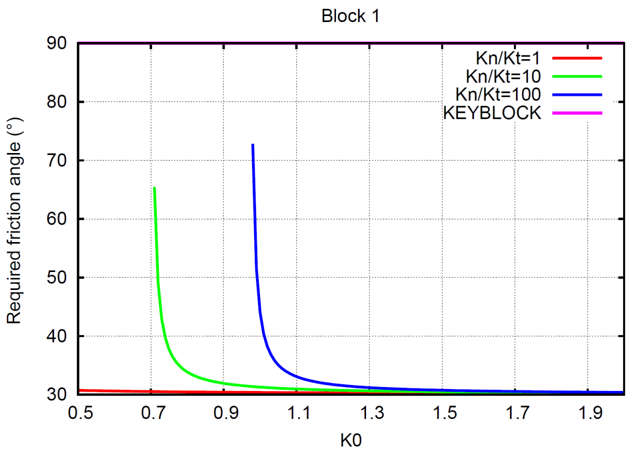

Fig. 7 Minimum friction angle required for stability, α=0°.

According to the ‘Keyblock’ method, the mode of displacement of this block is ‘free falling ’, thus it has a null safety factor, independently of the values of joint stiffnesses and initial in-situ stresses.

The analysis with ISOBLOC, shows that the block can be stable without support if the friction angle is high enough, (Fig. 7). For K_n/K_t=1, the increase of K_0 has no great effect on block’s stability. For example, for a friction angle of 32°, it is stable for all the values of K_0.

On the other hand, the increase of K_n/K_t has a destabilizing effect. However, for values of K_0 greater than 1.2 this effect is damped.

In this case, symmetry imposes the direction of movement to be vertical like in the ‘Keyblock’ method.

Block 2:

Rotating the block of 45° around the Z-axis will permit to study a flatter joint configuration.

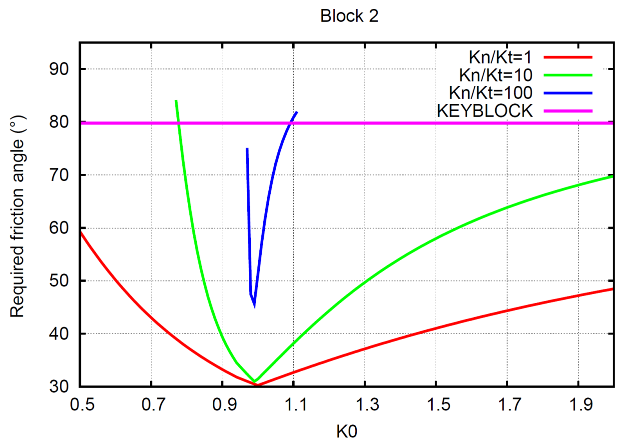

Fig. 8. Minimum friction angle required for stability, α=45°

It is shown in “Fig. 8” that elevated horizontal stresses do not necessarily contribute to the stability of the block. The change of curvature is due to the change in the direction of tangential stresses on the critical points. The case K_0=1 corresponds to the case of a hydrostatic stress field. Initial shear stresses are null on the block’s faces. It is the most stable state.

Like for the previous block, a low value of K_n/K_t contributes to the stability of block. “Fig. 9” shows, for a particular case of K_0=0.5, the distribution of final normal stress on the block’s faces for two values of K_n/K_t. The case of K_n/K_t=10 shows lower compression stresses as well as the apparition of tension. A high value of K_n/K_t has an effect of loosening of the block.

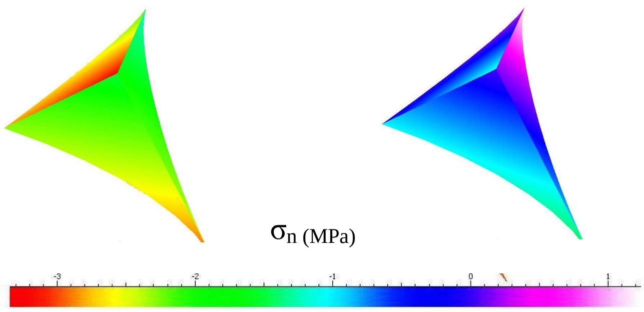

Fig. 9. Normal stresses for K_n/K_t=1 (left) and K_n/K_t=10 (right). Case α=45° and K_0=0.5.

Let’s note that the parameter ‘required friction angle’ cannot be calculated when tension occurs. This occurs for the case K_n/K_t=10 and more significantly for the case K_n/K_t=100 (Fig. 8). If a combination of K_n/K_t and K_0 results in tension on a point or area of the block, it is supposed to be unstable without support, whatever friction angle it has. ISOBLOC permits to calculate the minimum support needed to have stability. For example, for the case of a friction angle of 25º the required support pressure is plotted in “Fig. 10”.

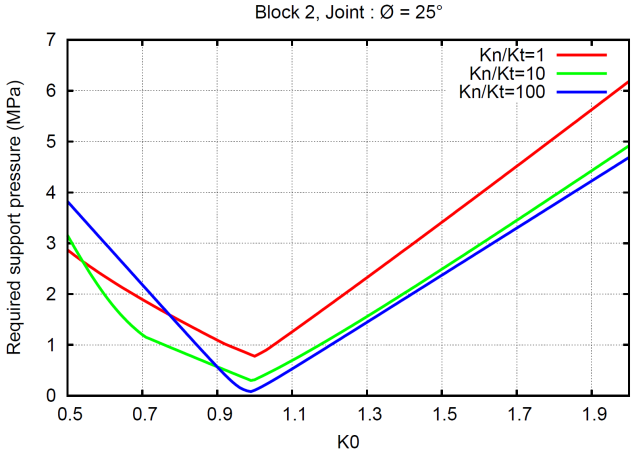

Fig. 10. Required support pressure needed for stability, α=45º

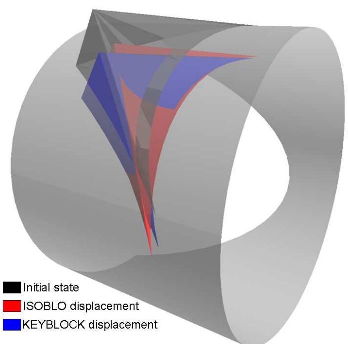

Fig. 11. Comparison of the normalized displacement vector of block 2 with the ‘Keyblock’ method and ISOBLOC (K_0=0.5 and K_n=K_t).

The ‘Keyblock’ analysis for that block results in a mode of displacement of sliding on one face (J2). The factor of safety is over conservative as compared to ISOBLOC method when only compression occurs.

“Fig. 11” shows the normalized displacement vector of block 2 obtained with ISOBLOC for K0=0.5 and K_n/K_t=1 as well as a comparison with the displacement obtained using ‘Keyblock’ method.

A rotation of the block is observed to the interior of the open space. Such a movement cannot be modelled with the ‘Keyblock’ method.

Block 3:
For high values of initial horizontal stresses, the friction angle required for stability is increased and tension occurs for the case of K_n/K_t=100 (See “Fig.12”).

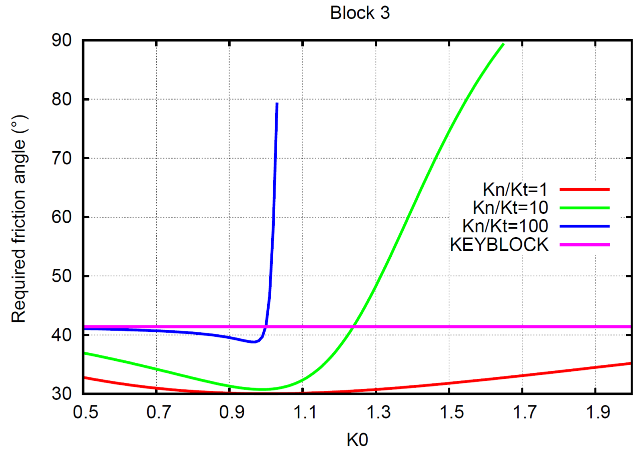

Fig. 12. Minimum friction angle required for stability, α=90º

In this case, the estimation of stability according to ‘Keyblock’ method is not conservative for all the combinations of K_0 and K_n/K_t. In fact, according to this theory, it is the direction of the active force (weight in this case), as compared to other discontinuities, that is determinant for the stability analysis. Thus, for the case of sliding on one joint, the more the joint is flat the more the block is stable. That explains why according to ‘Keyblock’, the block 3 is the most stable one

In reality, when a wedge is located underground, the effect of weight is negligible as compared to the effect of in-situ stresses. It is the intensities and direction of in-situ stresses as regards to the direction of joints that are the most determinant factors. According to ISOBLOC, results of blocks 1 and 3 are somehow equivalent: a high horizontal initial stress on block 3 gives quite equivalent results to a low horizontal initial stress on the block 1.

Rotation of principal stresses.
The block 2 studied is now rotated about the Y-axis at intervals of 1º, changing consequently the dip direction of the joints considered. Rotating the block in this way is also equivalent to considering different directions of principal stresses on one fixed block.

When K_{01}=K_{02}, the projection of the stress tensor on the block faces is the same. Thus, the following simulation treats the case of K_{01}=0.5 and K_{02}=1.

This kind of study can serve to determine the most critical direction of the principal stresses to use in design when information about these is not sufficient. (See “Fig.13”)

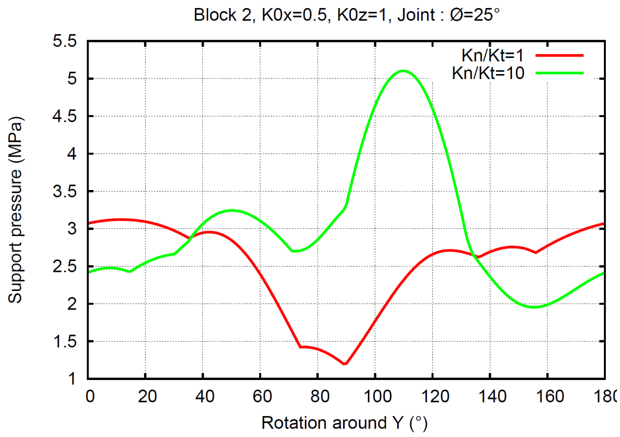

Fig. 13. Example of support pressure needed for various rotations of a block around Y-axis.

5. CONCLUSION AND PERSPECTIVES

A new numerical model to study isolated 3D rock blocks formed at the surface of underground excavation is presented. Comparing it to other isolated rock block methods, it is more complete and represents more rigorously the behavior of the underground blocks. In this model, in-situ stresses are included considering simultaneously rigid body movement in translation and rotation, rock joint behavior and equilibrium of forces and moments. The results obtained are the same as if a 3D finite elements model was applied to study the rock mass cut by an excavation with the presence of a single block on its surface, considering an infinite rigidity. Despite its accuracy and simplicity, this method has never been used in the domain of block mechanics.

The advantage of the algorithm developed is that any block geometry can be studied. The user just has to partition the complex faces into simple triangular or rectangular elements. Curved faces can be introduced by accurate meshing as well as blocks with concave faces.

Improving this newly developed approach would consist of including non linear rock joint mechanical behavior, considering hyperbolic normal behavior and elasto-plastic shear behavior.

This paper hasn't addressed the problem of validation of the concept of isolating the rock block and studying it separately. It has only focused on the development of an accurate model based on this concept. An interesting study would consist of performing a comparison of the ISOBLOC model with a full model that simultaneously considers the presence of all rock blocks.

6. AKNOWLEDGEMENTS

The authors would like to thank Rockscience for providing a free version of 'Unwedge' software.

REFERENCES

1. Goodman, R.E., and G.H. Shi. 1985. *Block theory and its application to rock engineering*. Englewood Cliffs, NJ: Prentice-Hall.

2. Warburton, P.M. 1981. Vector stability analysis of an arbitrary polyhedral rock block with any number of free faces. *Int. J. Rock Mech. Min. Sci. Geomech. Abstr.* 18:415–427.

3. Yeung, M. R., Q.H. Jiang, and N. Sun. 2003. Validation of block theory and three-dimensional discontinuous deformation analysis as wedge stability analysis methods. *Int. J. Rock. Mech. Min. Sci.* 40 : 265-275

4. Hatzor, Y., and R.E. Goodman. 1992. Application of block theory and the critical key block concept to tunnelling: two case histories. In: Myer LR, Cook NGW, Goodman RE, Tsang C (ed). *Proceedings of ISRM conference on fractured and jointed rock masses*. 663–70. Rotterdam: Balkema.

5. Warburton, P.M. 1983. A computer program for reconstructing blocky rock geometry and analyzing single block stability. *Computers and Geosciences*. 11(6):707-12

6. Windsor, C.R., and A.G. Thompson. 1992. SAFEX-A design and analysis package for rock reinforcement. *International Symposium on Rock Support*. Sudbury, Ontario.17-23

7. Starzec, P., and J. Andersson. 2002. Application of two-level factorial design to sensitivity analysis of keyblock statistics from fracture geometry. *Int. J. Rock. Mech. Min. Sci.* 39 (2): 243-255.

8. Liu, L., L. Zhongkui, and Z. Zhuoyuan. 2004. Stability analysis of block in the surrounding rock mass of a large underground excavation. Tunn. Und. Space Tech. 19(1): 35-44.

9. González-Palacio, C., A. Menéndez-Díaz, A.E. Álvarez-Vigil, and C. González-Nicieza. 2005. Identification of non-pyramidal key blocks in jointed rock masses for tunnel excavation. *Comp. Geotech.*. 32(3):179-200.

10. Yu, Q., Y. Ohnishi, G. Xue, and D. Chen. 2009. A generalized procedure to identify three-dimensional rock blocks around complex excavations. *Int. J. Num. Anal. Met. in Geomech.* 33(3):355-75.

11. Zhang, Y. M. Xiao, and J. Chen. 2010. A new methodology for block identification and its application in a large scale underground cavern complex. *Tunn. Und. Space Tech.* 25(2):168-80.

12. Theory manual for underground wedge stability analysis, Unwedge v3.0, Rocscience Inc., 2003

13. Hatzor, Y. , and A. Feintuch . 2005. The joint intersection probability. 2005. *Int. J. Rock. Mech. Min. Sci*. 42 (4):531-541.

14. Yarahmadi Bafghi, A.R. and T. Verdel . 2003. The key-group method. *Int. J. Num. Anal. Met. in Geomech*. 27(6): 495–511.

15. Curran, J.H., B. Corkum, and R.E. Hammah. Three dimensional analysis of underground wedges under the influence of stresses.

16. Lin, D. ,and C. Fairhurst. 1988. Static analysis of the stability of three-dimensional blocky systems around excavations in rock. *Int. J. Rock Mech. Min. Sci. Geomech. Abstr*. 25(3):139-147.

17. Mauldon M., and R.E. Goodman. 1996. Vector analysis of key block rotations. *J. Geotech Eng ASCE*. 122(12):976–987.

18. Fulvio Tonon, P.E. 1998. Generalization of Mauldon's and Goodman vector analysis of keyblock rotations. *J. Geo. Geoenv. Eng*. 913:922

19. Brady, B.H.G., and E.T. Brown. 1980. Excavation design in jointed rock. In *Rock Mechanics for Underground Mining*. 2nd ed. Chapman & Hall, 238-247

20. Crawford, A.M., and J.W. Bray. 1983. Influence of the in-situ stress field and joint stiffness on rock wedge stability in underground openings. *Can. Geotech. J. Toronto*. 20: 276-287.

21. Elsworth, D. 1986. Wedge stability in the roof of a circular tunnel: plane strain condition. *Int. J. Rock Mech. Min. Sci. Geomech. Abstr*. 23: 177-181.

22. Sofianos, A.I., P.P. Nomikos, and C.E. Tsoutrelis. 1999. Stability of symmetric wedge formed in the roof of a circular tunnel: non-hydrostatic natural stress field. *Int. J. Rock. Mech. Min. Sci*. 36: 687-691.

23. Nomikos, P.P., A.I. Sofianos, and C.E. Tsoutrelis. 2002. Symmetric wedge in the roof of a tunnel excavated in an inclined stress field. Int. *J. Rock Mech. Min. Sci*. 39: 59-67

24. Nomikos, P.P., P.V. Yiouta-Mitra, and A.I. Sofianos. 2006. Stability of asymmetric roof wedge under non-symmetric loading. *Rock Mech. Rock. Eng*. 39 (2):121-129.

25. Yow, J.L., and R.E. Goodman. 1987. A ground reaction curve based upon block theory. *Rock Mech Rock Eng*. 20:167-190.

26. Barton N., and S.C. Bandis. 1982. Effects of block size on the shear behaviour of jointed rocks. *23rd US Symposium on Rock Mechanic*.10 739–760. Rotterdam: Balkema.

27. Rechitskii V.I. 1998. Evaluation of the stiffness characteristics of rock joints from data of field observations at water-development projects. *Hydrotechnical construction*. 32 (8).

28. Tijani, M. VIPLEF. 2007. Notice d'utilisation. Centre de Géosciences, Mines Paris-Tech, Fontainebleau.